

International Journal of Pure, Applied and Computational Mathematics (IJPACM)

(An International Peer-Reviewed (Refereed) Journal)

Website- <https://ijpacm.nobleinkresearch.com/1/>

Email- editor@ijpacm.nobleinkresearch.com

ISSN Online: **2839-5875**; ISSN Print: **2839-5751**



Numerical study of MHD Boundary layer flow and Heat transfer in Ferrofluids past a Semi-Infinite flat plate

Harsimran Kaur^{1, *} and G.N. Verma^{2, *}

¹ Department of Applied Science, CEC-CGC, Landran, Mohali, Punjab (India)

² Department of Applied Sciences, SSIET, Derabassi, Mohali, Punjab (India)

*Corresponding author - simran1905@gmail.com, drvermagn@gmail.com

Received: 08 April 2026, Revised: 27 May 2026, Accepted: 17 June 2026, Available Online: 30 June, 2026

<https://doi.org/10.68219/grgxx710>

Abstract

The effect of heat transfer in Magneto hydrodynamics (MHD) in free convection flow of ferrofluids along horizontal flat plate is studied. For this purpose, magnetite Ferro particles (Fe_3O_4) are considered and water (H_2O) as the base fluid. In addition to these non-magnetic nanoparticles (Al_2O_3) were also used. The comparison between magnetite (Fe_2O_4) and non-magnetic (Al_2O_3) nanoparticles were conducted. The governing non-linear partial differential equations (PDE) were transformed into non-linear ordinary differential equations (ODE) using similarity transformation. The analytical solution were obtained for velocity and temperature gradient and solved with bvp4c MATLAB. The graphically results were plotted to show the behaviour of velocity and temperature of ferrofluids affected by parameters. It was observed that the temperature and velocity of ferrofluids depends upon viscosity and thermal conductivity together with magnetic field. The effect of rate of heat transfer, skin friction, Groshoff number and magnetic field parameters were also studied.

Keywords: Ferrofluids, Free convection, MHD

1. Introduction

Magneto Hydrodynamics (MHD) is the branch of continuum mechanics having study of magnetic properties and behavior of electrically conducting fluids. Many natural phenomenon and engineering problems are based upon the concept of MHD. The idea of MHD is that magnetic field induces current in conductive moving fluid, which creates forces on the fluid. The set of equations which describes the MHD are combination of Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. MHD has large applications in field of solar physics, plasma physics, biological flows etc. Heat transfer has huge applications in many manufacturing processes such as nuclear reactors, fuel cells, hybrid-powered engines, biomedical and pharmaceutical processes and pasteurization of food. In all above processes the heat transfer takes place through some heat transfer devices such as heat exchangers, evaporators, condenser and heat sinks.

Ferrofluids (FF) are some special of class of magnetic fluids with base liquids. They are also termed as Magnetic Nanofluids, particle size ranges from 5 to 15nm. In the absence of magnetic field, Ferrofluids acts like a liquid. A ferrofluids is composed of basically three major components; magnetic nanoparticles, base liquid and surface-active agent (Surfactant). The particles in ferrofluids are covered with surfactant; each of these particles behaves like permanent magnets. The surfactant protects the magnetic particles from congregating and falling out of suspension. Ferrofluids termed as colloidal suspension of Magnetite (Fe_3O_4), Hematite (Fe_2O_3) and other compound containing Iron and base liquids (usually oils). Fluids with magnetic properties are composed of two types of magneto rheological fluids (MR fluids) and ferrofluids. The major differences between two types fluids are size of magnetic fluids, in MR fluids particles are micron-sized while ferrofluids are comprised off nanoparticles. Due to smaller sized particles in ferrofluids are higher suspension stability and more suspension time. The composition of a typical ferrofluid is about 5% magnetic solids, 10% surfactant and 85% carrier, by volume.

Principle of ferrofluids is that keeps the atmosphere suspended above the earth, preventing it from collapsing under gravitational pull, to keep the tiny magnetic particles suspended. In addition, finite size of particles plays an important role. Due to smaller sized particles in ferrofluids are higher suspension stability and more suspension time. Ferrofluids are used in medical imaging; contrast agents/tracers for diagnosis via MRI and MPI. Ferrofluids has great applications in designing and operation of machine elements. Ferrofluids are used as liquid seals to keep dust out of computer harddrives. They can also be used in loudspeakers to help keep them cool. The thermal conductivity of ferrofluid is of much importance for researchers due to significant enhancement as compared to nonmagnetic nanofluids. In literature review it is observed that most of ferrofluids synthesized with magnetite (Fe_3O_4) which was produced by coprecipitation method has been studied. Abareshi and his coworkers synthesized ferrofluids by dispersing Fe_3O_4 nanoparticles in water (19). They observed an increase of 11.5% in the off-state thermal conductivity as the particle loading increased to 3 vol% at 40°C.

This increase was noticed when the magnetic field was applied parallel to the heat flux. Philip et al. and Shima et al. (5) investigated the thermal conductivity of kerosene-based ferrofluids synthesized with Fe_3O_4 nanoparticles. When the magnetic field was applied parallel to the heat flux, they discovered a substantial increase in the thermal conductivity. Heat transfer in ferrofluids using computational fluid dynamics techniques were studied by Jafari A & Sarmoma (3). The conceptual and experimental studies of ferrofluid flow and heat transfer are highly related to the magnetohydrodynamic (MHD) concept. Some more researchers initiate the works on MHD and hypothesized that if a magnetic field was constantly imposed to a conducting liquid, then the movement of the liquid induced the electromagnetic force and produced electric currents which in turn changed the state of motion of the liquid. Gul et al. (11) expanded that the polarization of magnetite (Fe_3O_4) in the magnetic field direction was very strong and it kept polarized even the magnetic field was abolished. The research of MHD rises from its enormous applications in petroleum industries, geophysical problem, and agricultural engineering. The interaction of electrically conducting liquid and magnetic field strongly affects industrial equipment like bearings, MHD generators and pumps. MHD concept also can be utilized in medical sector such as tumors elimination, gastric medications, wound treatment and sterilization of devices. The effect of magnetic field on the convection of nanofluid also has been investigated for various conditions and circumstances by using the Galerkin method. Mukhopadhyay (9) investigated MHD boundary layer flow with the presence of thermal stratification and concluded that the higher magnetic parameter suppressed the velocity field and subsequently enhanced the skin friction coefficient. Later on Nayan, Zakaria and their co-workers studied MHD flow of hybrid nanofluids over vertical plate in porous media. Khan WA& Khan ZH (5) studied MHD boundary layer flow of nanofluid past vertical plate.

According to above literature review it can be noticed that more studies were focused on free convection flows in nanofluids rather than ferrofluids. This study focused on heat transfer in free convection flows along flat plate in porous medium. In this problem the comparison between nonmagnetic(Al_2O_3) nanoparticles and magnetic (Fe_2O_3) are also conducted. The study investigates MHD free convection of ferrofluids in vertical plate immersed in porous medium. The temperature also kept constant and maintained no slip condition. From study the effects of magnetic field, skin friction, dimensionless temperature, Grashof number, heat transfer and porous medium parameters are evaluated.

2. Mathematical Formulation

In the formulation of the problem, the water based ferrofluid (Fe_2O_3) considered for the study in free convection along flat plate immersed in porous media. The effect of radiation in free convection was also studied. The aligned magnetic field with an acute angle α , applied to fluid. It is expressed as function of distance from origin expressed as $B(x) = \frac{B_0}{\sqrt{x}}$ with B_0 not equal to zero, B_0 magnetic field strength and x is the coordinate along the plate. When magnetic field is

induced, ferroparticles get oriented along magnetic field lines. The surface of plate is maintained at constant wall temperature T_w and T_∞ free stream temperature.

Taking into consideration of above assumptions and based upon approximation of Boussinesq, the governing equations of momentum and energy are as follows:

$$\rho_{ff} \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \sigma_{ff} B_0^2 u + (\rho\beta)_{ff} g(T - T_0) + \mu_{ff} \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$(\rho c_p)_{ff} \frac{\partial T}{\partial y} = k_{ff} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y} \quad (2)$$

with boundary conditions

$$u(0, t) = 0, \quad T(0, t) = T_0 \quad (3)$$

$$u(d, t) = 0, \quad T(d, t) = T_w \quad (4)$$

where the subscript ff stands for ferrofluids. $u = u(y, t)$ fluid velocity in x-direction, $T = T(y, t)$ is the ferrofluid temperature, ρ_{ff} is ferrofluid density, μ_{ff} is dynamic viscosity of ferrofluid, σ_{ff} is electrical conductivity of ferrofluid, q is radiative heat flux in x-direction, g is acceleration due to gravity, $(\rho c_p)_{ff}$ specific heat capacity at constant pressure, $(\rho\beta)_{ff}$ is thermal expansion of ferrofluid.

With reference to [16, 17, 19] the relations of density, dynamic viscosity and thermal conductivity of ferrofluids with base fluid are as follows:

$$\rho_{ff} = (1 - \phi)\rho_f + \phi\rho_s, \mu_{ff} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

$$(\rho\beta)_{ff} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s, \alpha_{nf} = \frac{\sigma_{ff}}{(\rho C_p)_{ff}}$$

$$(\rho C_p)_{ff} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$$

$$\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (5)$$

Where μ_f – dynamic viscosity of the base fluid, ϕ – solid volume fraction, ρ_f – density of base fluid, ρ_s – density of solid nanoparticles, $(\rho C_p)_{nf}$ – heat capacity of ferrofluid and k_{ff} – thermal conductivity of ferrofluid, Where k_f & k_s – thermal conductivities of the base fluid and ferroparticles. $(\rho C_p)_f$ & $(\rho C_p)_s$ are the specific heat parameters of base fluid and ferroparticles.

Based on Makinde and Mhone [9], the radiative heat flux is as follows:

$$-\frac{\partial q}{\partial y} = 4\alpha_0^2 (T - T_0)$$

And pressure gradient is given by (6)

$$-\frac{\partial p}{\partial x} = \lambda \exp(iw_1 t)$$

α_0 mean radiation absorption, λ is constant and w_1 is oscillation frequency.

Introducing non-dimensional variables:

$$x^* = \frac{x}{d}, y^* = \frac{y}{d}, u^* = \frac{u}{U_0}, t^* = \frac{tU_0}{d}, p^* = \frac{d}{\mu_f U_0} p, T^* = \frac{T - T_0}{T_w - T_0}, w^* = \frac{dw_1}{U_0} \quad (7)$$

The system of equations (1) to (4) reduces to:

$$a_0 \frac{du}{dt} = \lambda \exp(iwt) + \phi_2 \frac{d^2 u}{dy^2} - a_3 u + a_1 T \quad (8)$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0 \quad (9)$$

$$b_0^2 \frac{dT}{dt} = \frac{d^2 T}{dy^2} + b_1^2 T \quad (10)$$

$$T(0, t) = 0, \quad T(1, t) = 0, \quad t > 0 \quad (11)$$

Where

$$\begin{aligned} a_0 &= \phi Re, a_1 = \phi_3 Gr, Re = \frac{U_0 d}{\nu}, \phi_1 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f}, \phi_2 = \frac{1}{(1 - \phi)^{2.5}} \\ \phi_3 &= (1 - \phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f}, \phi_4 = (1 - \phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}, M^2 = \frac{\sigma_{ff}}{\mu_{ff}} B_0^2 d^2, Gr = \frac{g\beta_f d^2 (T_w - T_0)}{\nu_f U_0} \\ N^2 &= \frac{4d^2 \alpha_0^2}{k_{ff}}, Pe = \frac{(\rho c_p)_f}{k_f}, \lambda_n = \frac{k_f}{k_{ff}}, b_1 = \sqrt{\frac{N^2}{\lambda_n}}, b_0 = \frac{\phi_4 Pe}{\lambda_n} \end{aligned}$$

Here M , Re , N , Pe , and Gr denote magnetic parameter, Reynold's number, radiation parameter, Peclet number and thermal Grashof number, respectively. Other arbitrary constants were used for clarity and simplification.

3. Method of Solution

Equations (8) to (11) are not easy to solve, so use perturbation method which is quite suitable to solve these equations, the parameter ε is assumed to small such that $0 < \varepsilon < 1$

$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t} \quad (12)$$

$$T(y, t) = T_0(y) + \varepsilon T_1(y) e^{i\omega t} \quad (13)$$

Using equations (12) and (13) into equations (8) to (11) converted into ordinary differential equations, after large calculations we get

$$T(y, t) = \frac{\sin b_1 y}{\sin b_1} \quad (14)$$

$$u(y, t) = \frac{(a_2 + b_1^2 + m_1^2)}{\sinh m_1 (b_1^2 + m_1^2)} \sinh m_1 y - \frac{a_2 \sin b_1 y}{(b_1^2 + m_1^2) \sin b_1} + \varepsilon e^{i\omega t} \left[\frac{\lambda (\cosh m_2 - 1)}{m_2^2 \phi_2 \sinh m_2} \sinh m_2 y - \frac{\lambda \cosh m_2 y}{m_2^2 \phi_2} + \frac{\lambda}{m_2^2 \phi_2} \right] \quad (15)$$

Skin friction and Nusselt number are evaluated from equations (14) and (15)

$$Nu = \frac{b_1}{\sin b_1} \quad (16)$$

$$c_f = \frac{(a_2 + b_1^2 + m_1^2)}{\sinh m_1 (b_1^2 + m_1^2)} m_1 - \frac{a_2 b_1}{(b_1^2 + m_1^2) \sin b_1} + \varepsilon e^{i\omega t} \left[\frac{\lambda m_2 (\cosh m_2 - 1)}{m_2^2 \phi_2 \sinh m_2} \right] \quad (17)$$

According to Turkyilmazoglu [20], the relations in Eq (5) are restricted to ferroparticles with spherical shape. For ferrooparticles of other shapes with different thermal conductivity and dynamic viscosity would be modified.

Table1: Spherical & Cylindrical shaped ferroparticles with dynamic viscosity & thermal conductivity:

S.no	Shape of ferroparticles	Dynamic Viscosity	Thermal Conductivity
1	Spherical	$\mu_{ff} = \frac{\mu_f}{(1-\phi)^{2.5}}$	$\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$

2	Cylindrical	$\mu_{ff} = \frac{\mu_f}{(1-\phi)^{2.5}}$	$\frac{k_{ff}}{k_f} = \frac{k_s + \frac{1}{2}k_f - \frac{1}{2}\phi(k_f - k_s)}{k_s + \frac{1}{2}k_f + \phi(k_f - k_s)}$
---	-------------	---	---

Table 2: Thermo physical properties of ferroparticles(Fe₃O₄ & Al₂O₃) and base fluid:

Physical Properties	Water	Fe ₃ O ₄	Al ₂ O ₃
ρ (kg / m ³)	997.1	5180	3970
c _p (J/kgK)	4184	670	765
k (W/mk)	0.613	9.7	40
β x 10 ⁻⁵ (K ⁻¹)	21	0.5	0.85

Results & Discussion:

Figure 1: Velocity graph for $N = 0.1, 0.9, 1.2, 1.4$ when $Gr = 0.1, \phi = 0.04, Re = 1, Pe = 1, M = 1, \lambda = 1, t = 5$ and $\omega = 0.2$.

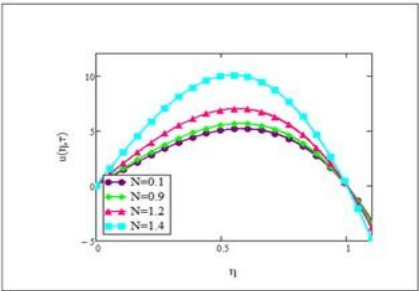
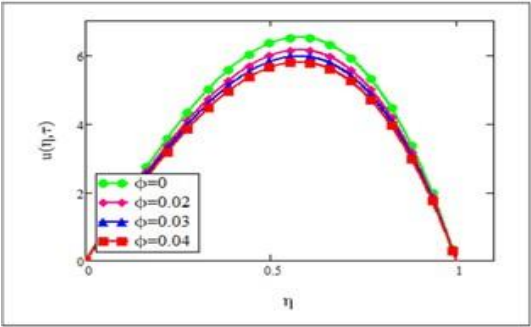


Figure2: Velocity graph for $\phi = 0.0, 0.02, 0.03, 0.04$ when $Gr = 0.1, N = 1, Re = 1, Pe = 1, M = 1, \lambda = 1, t = 5$ and $\omega = 0.2$.



Figures 3: Velocity graph for $M = 0, 2, 4, 5$ when $Gr = 0.1$, $\phi = 0.04$, $Re = 1$, $Pe = 1$, $N = 1$, $\lambda = 1$, $t = 5$ and $\omega = 0.2$.

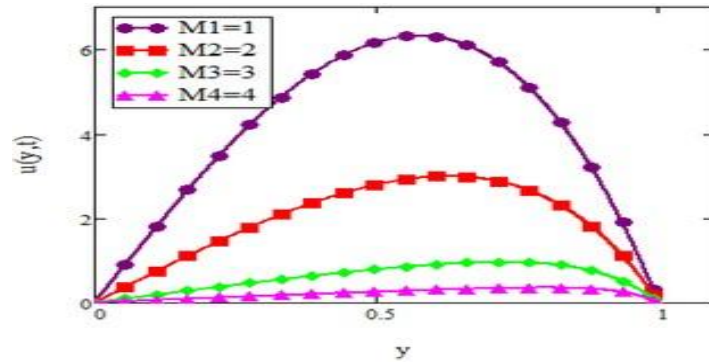


Figure 4: Comparison of ferrofluid (Fe_3O_4) velocity with nanofluid (Al_2O_3) velocity when $Gr = 0.1$, $\phi = 0.04$, $Re = 1$, $Pe = 1$, $M = 1$, $\lambda = 1$, $\tau = 5$ and $\omega = 0.2$.

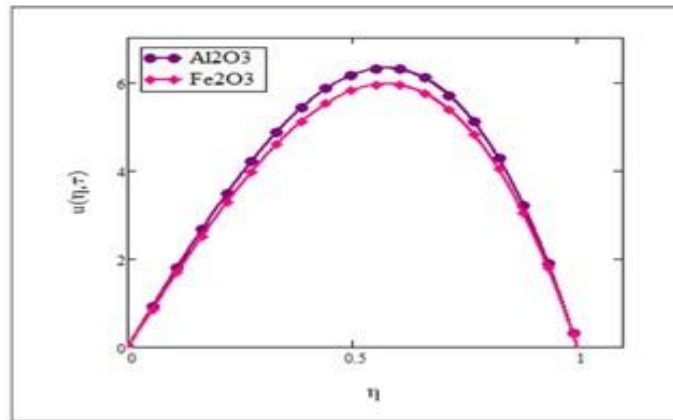


Figure 5: Temperature graph for ϕ when $N = 1.5$ and $t = 0.5$.

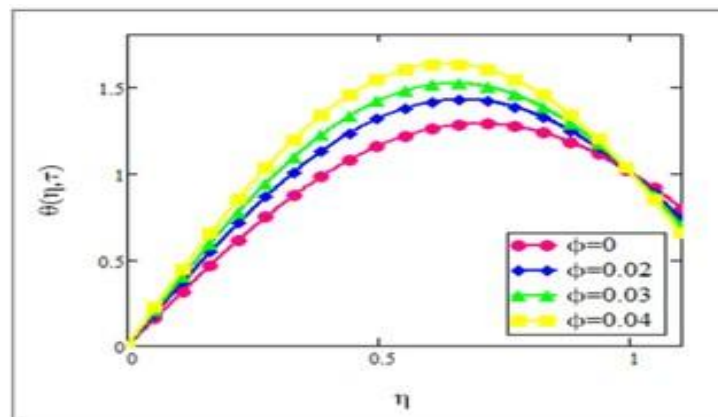
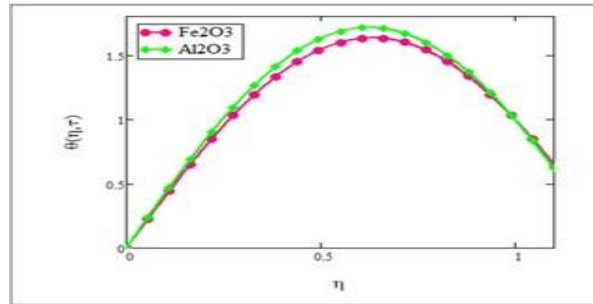


Figure 6: Comparison of ferrofluid (Fe_3O_4) temperature with nanofluid (Al_2O_3) temperature when $\phi = 0.04$, $N = 1.5$ and $\tau = 0.5$.



Conclusions:

- From figure 1, it is noticed that the velocity of ferrofluid decreases with increase in the volume fraction of nanoparticles, because of increase in the viscosity.
- There is effect of radiation parameter N on the velocity of ferrofluid, due to decrease in viscosity, velocity of ferrofluid increases.
- Different values of magnetic parameter M for velocity profile were considered. An increase in magnetic parameter M led to decrease in velocity of ferrofluid.
- Comparison of velocities when two different types of nanoparticles namely, magnetite (Fe_3O_4) and non-magnetic (Al_2O_3) were added to the conventional base fluid. It was found that the velocity of the Al_2O_3 nanofluid was greater than the velocity of the ferrofluid (Fe_3O_4 nanofluid) when the volume fraction was $\phi = 0.04$. This shows that the viscosity and thermal conductivity of the ferrofluid was greater than the Al_2O_3 nanofluid.

References

1. Abareshi, M., Goharshadi, E. K., Zebarjad, S. M., Fadafan, H. K., & Youssefi, A. (2010). Fabrication, characterization and measurement of thermal conductivity of Fe_3O_4 nanofluids. *Journal of Magnetism and Magnetic Materials*, 322(24), 3895–3901. <https://doi.org/10.1016/j.jmmm.2010.07.036>
2. Asma, K., Khan, I., & Sharidan, S. (2015). Exact solutions for free convection flow of nanofluids with ramped wall temperature. *The European Physical Journal Plus*, 130, 57. <https://doi.org/10.1140/epjp/i2015-15057-6>
3. Blasius, H. (1908). Grenzsichten in Flüssigkeiten mit kleiner Reibung. *Zeitschrift für Mathematik und Physik*, 56, 1–37.
4. Brinkman, H. C. (1952). The viscosity of concentrated suspensions and solutions. *The Journal of Chemical Physics*, 20(4), 571–581. <https://doi.org/10.1063/1.1700493>
5. Gul, A., Khan, I., & Shafie, S. (2018). Radiation and heat generation effects on MHD mixed convective flow of nanofluids. *Thermal Science*, 22(Suppl. 1A), S51–S62. <https://doi.org/10.2298/TSCI160620299G>

6. Haq, R. U., Nadeem, S., Khan, Z. H., & Noor, N. F. M. (2015). Convective heat transfer in MHD slip flow over a stretching surface in the presence of carbon nanotubes. *Physica B: Condensed Matter*, 457, 40–47. <https://doi.org/10.1016/j.physb.2014.09.013>
7. Jafari, A., Tynjälä, T., Mousavi, S. M., & Sarkomaa, P. (2008). Simulation of heat transfer in a ferrofluid using computational fluid dynamics technique. *International Journal of Heat and Fluid Flow*, 29(4), 1197–1202. <https://doi.org/10.1016/j.ijheatfluidflow.2008.03.001>
8. Khan, W. A., Khan, Z. H., & Haq, R. U. (2015). Flow and heat transfer of ferrofluids over a flat plate with uniform heat flux. *The European Physical Journal Plus*, 130, 86. <https://doi.org/10.1140/epjp/i2015-15086-7>
9. Khan, Z. H., Khan, W. A., Qasim, M., & Shah, I. A. (2014). MHD stagnation-point ferrofluid flow and heat transfer toward a stretching sheet. *IEEE Transactions on Nanotechnology*, 13(1), 35–40. <https://doi.org/10.1109/TNANO.2013.2290005>
10. Makinde, O. D. (2011). Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition. *Thermal Science*, 15(Suppl. 1), S137–S143. <https://doi.org/10.2298/TSCI100415036M>
11. Makinde, O. D., & Mhone, P. Y. (2005). Heat transfer to MHD oscillatory flow in a channel filled with porous medium. *Romanian Journal of Physics*, 50(9–10), 931–938.
12. Mehta, R., Chouhan, V. S., & Mehta, T. (2020). MHD flow of nanofluids in the presence of porous media, radiation and heat generation through a vertical channel. *Journal of Physics: Conference Series*, 1504, 012008. <https://doi.org/10.1088/1742-6596/1504/1/012008>
13. Mojumder, S., Saha, S., Saha, S., & Mamun, M. A. H. (2015). Effect of magnetic field on natural convection in a C-shaped cavity filled with ferrofluid. *Procedia Engineering*, 105, 96–104. <https://doi.org/10.1016/j.proeng.2015.05.012>
14. Philip, J., Shima, P. D., & Raj, B. (2007). Enhancement of thermal conductivity in magnetite-based nanofluid due to chain-like structures. *Applied Physics Letters*, 91(20), 203108. <https://doi.org/10.1063/1.2812699>
15. Qasim, M., Khan, Z. H., Khan, W. A., & Shah, I. A. (2014). MHD boundary layer slip flow and heat transfer of ferrofluid along a stretching cylinder with prescribed heat flux. *PLOS ONE*, 9(1), e83930. <https://doi.org/10.1371/journal.pone.0083930>
16. Rana, P., Bhargava, R., & Beg, O. A. (2012). Numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium. *Computers & Mathematics with Applications*, 64(9), 2816–2832. <https://doi.org/10.1016/j.camwa.2012.02.042>

17. Reddy, P. S., Chamkha, A. J., & Al-Mudhaf, A. (2017). MHD heat and mass transfer flow of a nanofluid over an inclined vertical porous plate with radiation and heat generation/absorption. *Advanced Powder Technology*, 28(3), 1008–1017. <https://doi.org/10.1016/j.appt.2016.12.023>
18. Rosensweig, R. E. (1985). *Ferro hydrodynamics*. Cambridge University Press.
19. Sheikholeslami, M., & Ganji, D. D. (2014). Ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow and convective heat transfer. *Energy*, 75, 400–410. <https://doi.org/10.1016/j.energy.2014.07.089>
20. Turkyilmazoglu, M. (2014). Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer. *Journal of Heat Transfer*, 136(3), 031704. <https://doi.org/10.1115/1.4026072>
21. Ilias, M. R., Rawi, N. A., & Shafie, S. (2017). Steady aligned MHD free convection of ferrofluids flow over an inclined plate. *Journal of Mechanical Engineering*, 14(2), 1–15.